Critical Density of the Universe

Niemack Lab, July 1, 2016

Let's derive the critical density of the universe using only our knowledge of kinetic and potential energy and the Hubble's law. Hubble's law states a linear relation between the recessional velocity and the proper distance of an observed object. Assume the following equations:

Kinetic Energy $= \frac{1}{2}mv^2$
Gravitational Potential Energy $= -\frac{GMm}{r}$
Hubble's Law: $v = Hr$

(a) Using your knowledge of kinetic and potential energy, write down the escape velocity of an object – say, for our scenario, one galaxy escaping from all the rest of the universe. Recall that the escape velocity is the minimum speed that an object needs to have in order to escape the gravitational attraction of a certain massive body. Express your result in terms of G, M and r.

(b) Model the universe from which the galaxy is escaping as a uniform sphere, and rewrite (a) in terms of ρ, the density of the universe, G, r, and mathematical constants.

(c) Use your equation from (b) along with Hubble's Law to write down the critical density ρ_c of the universe in terms of H, G, and mathematical constants. If the density of the universe is greater than the density you've written down, the universe will contract, but if it is less, it will expand forever!

(d) By plugging in numerical values for H and G, estimate ρ_c in units of kg/m^3. Using the mass of a proton, how many protons per cubic meter of space does ρ_c correspond to? What is the average density of our universe today, and what does your calculation tell us about the evolution of our universe? What role do dark matter and dark energy play in determining whether our universe will expand or contract?